

## AD-3258

## Third Year B. Sc. (Sem. VI) Examination March/April - 2015

## Fourier Transform & its Applications

(Generic Elective - 6004)

Time: 3 Hours]	[Total Marks: 70
Instructions:	
(1)	<b></b>
નીચે દર્શાવેલ 👉 નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.	Seat No.:
Fillup strictly the details of - signs on your answer book.  Name of the Examination:	
Third Year B. Sc. (Sem. VI)	
Name of the Subject :	/ /ال
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FOURIER TRANSFORM & ITS APPLICATIONS	]
Section No. (1.2): Nil	Student's Signature

- (2) All questions are compulsory.
- (3) Figures to the right indicate full marks of the corresponding question.
- 1 Answer the following: (any five)

**5** 

- (1) State Parseval's identities for Fourier cosine transform.
- (2) State one dimensional heat-flow equation.
- (3) Write the complex form of Fourier Integral and also write inverse Fourier Transform of f(x).
- (4) Define Convolution of two functions.

(5) Prove that 
$$F\left[\frac{\partial^2 u}{\partial x^2}\right] = -s^2 F[u]$$
.

- (6) State the change of scale property for Fourier sine transform.
- (7) Define Fourier transform.
- (8) State one dimensional wave equation for vibrating string.

 $\mathbf{2}$ State and prove shifting property of Fourier (a) Transform.

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OR

Derive Fourier sine and cosine integral. (a)

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(b) Attempt any two: 7

(1) Solve the integral equation

$$\int_{0}^{\infty} f(\theta) \cos \alpha \theta d\theta = \begin{cases} 1 - \alpha; & 0 \le \alpha \le 1 \\ 0; & \alpha > 1 \end{cases}$$
. Hence evaluate

$$\int_{0}^{\infty} \frac{\sin^2 t}{t^2} dt = \frac{\pi}{2}$$

Find the Fourier transform of  $f(x) = \begin{cases} 1, |x| < 1 \\ 0, |x| > 1 \end{cases}$ . (2)

Hence evaluate 
$$\int_{0}^{\infty} \frac{\sin x}{x} dx$$
.

3 (a) State and prove convolution theorem for Fourier Transform.

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OR

State parseval's identity for Fourier Transform and (a) prove it.

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(b) Attempt any two:

Using parseval's identity, prove that

$$\int_{0}^{\infty} \frac{dt}{\left(a^2 + t^2\right)\left(b^2 + t^2\right)} = \frac{\pi}{2ab\left(a + b\right)}$$

Using Parseval's identity, If  $f(x) = \begin{cases} 1; & 0 < x < 1 \\ 0; & x > 1 \end{cases}$  then

evaluate 
$$\int_{0}^{\infty} \frac{\sin^2 x}{x^2} dx$$
.

4 (a) Derive Fourier transform of the derivatives of a function.

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OR

- (a) Determine the distribution of temperature in the semi 8 infinite medium  $x \ge 0$ , when the end x = 0 is maintained at zero temperature and the initial distribution of temperature is f(x).
- (b) Attempt any two:

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(1) Using the Method of residues, evaluate

$$L^{-1}\left\{\frac{1}{s^2\left(s^2-a^2\right)}\right\}.$$

(2) Using the method of residues, evaluate

$$L^{-1}\left\{\frac{1}{(s+1)(s-1)^2}\right\}.$$